The synergy effect of stochastic dominance and value-at-risk

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ABSTRACT

The article provides special criterion to choose a group of superior corporations and points out a link between Stochastic Dominance (SD) and Excess Return (ER). The criterion includes dual assessing: loss risk and profitability. In practice, our criterion can capture the advantages of Value at Risk (VaR) and SD respectively. This choice rule is called 'SD-VaR criterion' and a corporation is pointed out by this criterion will not have extreme risk or profit. These corporations also have the excess returns and persistency of its good status.

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1. Introduction

This article tries to construct a classification principle that simplifies the choice-space and eliminates the unwanted noise. An investor selects her or his portfolio and usually stresses two factors: the risk and the profitability. If the fact that a portfolio has high return with high risk of loss holds, the choice criterion we created will provide more profitability chance with less risk. In other word, the lowest risk of loss or the highest return is not a best choice and also not included to our choice set. This article's purpose is that makes an investor can improve or increase their opportunities to profit with a lower risk using our criterion.

There are many financial distresses or default analysis on the literatures, but a few articles to investigate conditions of the healthy corporations. If we know a group of superior corporations with excess returns and perform a simple portfolios has equal weight on those corporations, the return of this strategy will be more than normal return. An investor can be easier to collect the return data of a financial commodity than other variables on the accounting reports. Therefore, we develop a criterion to point out these good corporations, and this filter rule just includes a return variable and can be employed on most financial instruments.

Bawa (1978), Dentcheva and Ruszczynski (2006), Davidson and Duclos (2000), Kopa and Post (2009), Post (2003) and so on, they consider that stochastic dominance (SD) is a good rule to assess the chance to profit for a corporation. Another, the value at risk (VaR) or Condition-VaR (CVaR) is a convention principle on the literatures to determine the risk of loss. But, the two important rules: SD and VaR have not integrated or formed a criterion to help people select an optimal portfolio. It's a pity that there is no paper extends to their grand contributions. Therefore, we plan to investigate and create a new criterion that includes the advantages of these two

popular rules.

All of SD empirical studies on the literatures have a difficult on a large number of computing. It is a reason to lead to a few papers operates and compares overall SD relationship for all corporations in the twentieth century. Due to SD has no completeness and cannot sort all corporations, and the first step we find the SD relationships between one and remain other and more than 5000 corporations.[1] This step needs to calculate more than 5000 probability density functions (PDFs) and cumulative density functions (CDFs) for all corporations and there are more than 20 domains for a function. The next step will repeat more than 5000 runs the SD comparison on the first step. That is enormous amount of computation and therefore we code them using MATLAB and resolve the problem on the stochastic dominance works.[2]

2. Description of the targets

A Multiple filter is better than single one and we plan to aggregate their superiorities of every filter we include. If a difference between A and B criterion is obviously, the complementary effect of the multiple filter is much larger. This is one of central ideas in this article and we adopt two comprehensive and fundamental filters to build a new criterion. VaR and SD are the basic tools and corresponds risk and profitability factor respectively. The common element is rate of return and uses it to link these advantages of these two filters.

A. Definitions of VaR and SD

We refer to the ordinary definitions of VaR in Duffie and Pan (1997) and

[1] The definition of completeness can refer to Mas-colell *et al.* (1997). The rating of corporation sorted by VaR is a supplement to the incompleteness of SD.

[2] We show MATLAB codes and data on Lin's website after publication.

Rockafellar and Uryasev (2002), they consider that VaR is the α critical value of probability distribution of changes in market value. In other world, VaR is the domain makes cumulative probability function [CDF, $\Phi(\cdot)$] of loss equal to α and can be expressed as,

$$\eta^{+}(\alpha) = inf\{\ \eta \mid \Phi(x,\eta) > 1-\alpha\ \} \tag{1}$$
$$= VaR(\alpha, x)$$

Where we give some technical assumptions and denote that

$$\Phi(x,\eta) = Pr(y|Loss(x,y) \leq \eta) \tag{2}$$

, y is a random scalar, x is a decision and η is a negative rate of return. The VaR in our article is expressed as a negative rate of return since every corporation's size is selfsame. Therefore, the value of a loss dividing by corporation's scale is suitable to measure VaR.

Mathematical definition of *first-order stochastic dominance* (FSD) in Mas-Colell, Whinston and Green (1995) is: the distributions $F_A(\cdot)$ *first-order stochastically dominates* $F_B(\cdot)$ for every non-decreasing function u: R→R and can represent that

$$\int u \cdot F_A(y)\ \geq\ \int u \cdot F_B(y)\,. \tag{3}$$

In order to apply and operate the FSD in this article, FSD will become the discrete type:

$$F_A(y) \leq F_B(y)\ \ for\ all\ y \tag{4}$$

and there is at least one y_i such that $F_A(y) < F_B(y)$.

The investment A has the *second-order stochastic dominance* (SSD) over investment B if the former is more predictable and has at least as high a mean. In a word, the cumulative value of investment A's CDF is greater than or equal to investment B's cumulative value and we also make SSD to rewrite

$$\sum_{-\infty}^{y}[F_B(y)] \leq \sum_{-\infty}^{y}[F_A(y)] \quad for\ all\ y \tag{5}$$

and there is at least one y_i such that $\sum_{-\infty}^{y_i}[F_A(y)-F_B(y)] < 0$.

Let F_A and F_B be the CDFs of two distinct investments *A* and *B*. *A third-order stochastic dominances B* (TSD) if and only if,

$$\sum_{-\infty}^{y}\{\ \sum_{-\infty}^{\infty}[F_A(x)-F_B(x)]\ \} \leq 0 \quad for\ all\ y \tag{6}$$

and there is at least one y_i such that $\sum_{-\infty}^{y_i}\{\ \sum_{-\infty}^{\infty}[F_A(y)-F_B(y)]\ \} < 0$.

Equivalently, *A* dominates *B* in the third order if and only if $E_A(y) \geq E_B(y)$ for all non-decreasing, concave utility functions *U* those are positively skewed. The original TSD definition can refer to Whitmore (1970). Bawa (1975) shows that TSD is the optimal rule when comparing uncertain outcomes with equal first-moments.

B. The Hit-ratio of SD-VaR criterion

There are more than five thousands corporations in our dataset, but we just include 2805 corporations in the analysis. That's since the SD is computed by CDF of returns and the analysis chooses a corporation has more than 100 returns in COMPUSTAT database. According to the limit theory, the outcomes CDF we obtain are robust and

stable using the number of returns for a corporation is more than 100. Then, we rank these 2805 corporations by their VaR(α =0.1):

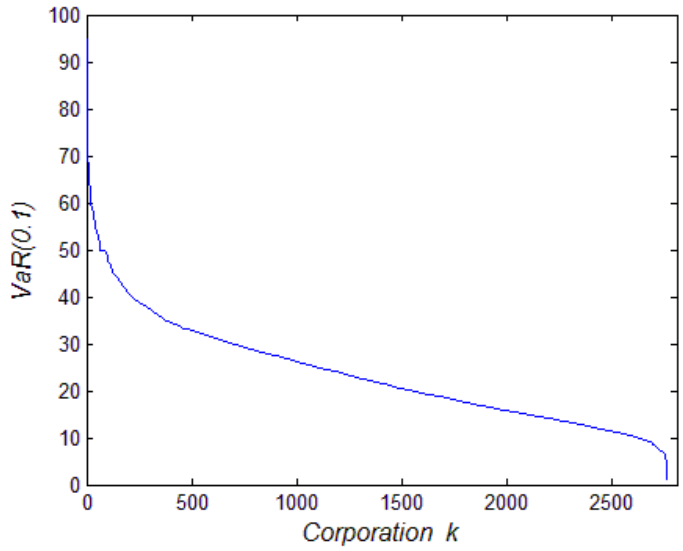

Figure 1. The corporation rank by *VaR* (α =0.1).

The VaR are computed by the past absolute loss of a corporation and we sort and rank all corporations from zero to 2804. A corporation ranked 2804 has a minimum risk or ranked zero has the highest risk than others. Next, we want to find the relationships between VaR and the probability of satisfied our criterion. The lower VaR corporation seems that should easy to stochastic dominate other corporation on the intuitions. The values of VaR can be sorted, but the SD has not a completeness property and sometime no corporation can stochastic dominate another one. Therefore, we cannot sort them using SD criterion, however a probability theory is a suitable tool to capture the SD features. These SD are based on the probabilities and we will give a proof that the SD can measure a corporation's chance to profit.

Ma and Wong (2010), they have a theorem to convey the relationship between SD and VaR. They say that

$$A \overset{FSD}{\succcurlyeq} B \qquad equivalent\ to \quad VaR(\alpha, A) \leq VaR(\alpha, B) \tag{7}$$

,but this inference seem not always to be satisfied. Therefore we adopt a probability to examine the existence of theorem and the possibility of SD-VaR criterion held is a hitting ratio and we express that,[3] where $i \in \{1, 2, \cdots, N\}$, k is the base corporation for the VaR and $N[\cdot]$ is the number of corporations that satisfies those special conditions. This base corporation k could compare his VaR and SD feature with all corporations' and obtains a hit-ratio of SD-VaR criterion. On the literatures, there are three orders of SD and we check whether those SDs have the different trend of hit-ratios. These hit-ratios are showed in following table:[4]

The X axis of Figure 2 is the same Figure 1's and their values (1,2,…, 2805) response the VaR rank of all corporations. The higher values of X axis express the lower risk of loss and the first assess VaR criterion can show in this design of Figure 2. We observe these three hit-ratios of SD-VaR criterion and preliminary consider that they are closed. Three *p*-values of their differences are all greater than 0.1 and we also show them on a figure.

[3] To simplify the notations, we use $VaR(i)$ to instead of $VaR(\alpha, i)$.
[4] These hit-ratios are produced by dataset includes Dec/2011 returns.

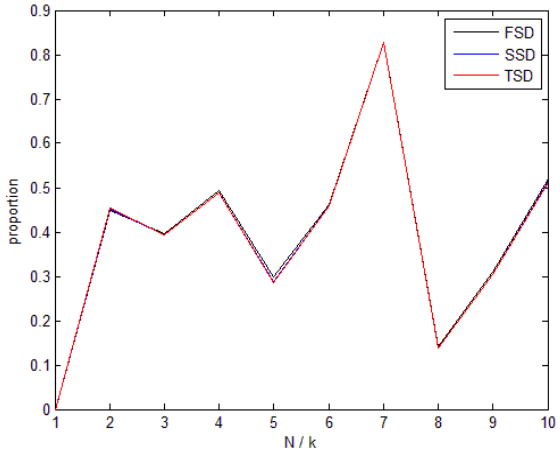

Figure 2. The three SD-trends of hit-ratios for 10 corporations.

Every hit-ratio is computed by 2805 SD relationships of these 10 corporations and almost doesn't have a problem in sample size. According to above table and figure, we seem to make sure that three kinds of SD have the similar path. Furthermore, we include all corporations and compute every corporation's hit-ratio to examine the argument that three SDs have the same trend of hit-ratios. Therefore, can we draw some findings from the special configuration in those diagrams? The answer is 'yes' and they would be extended for all corporations.

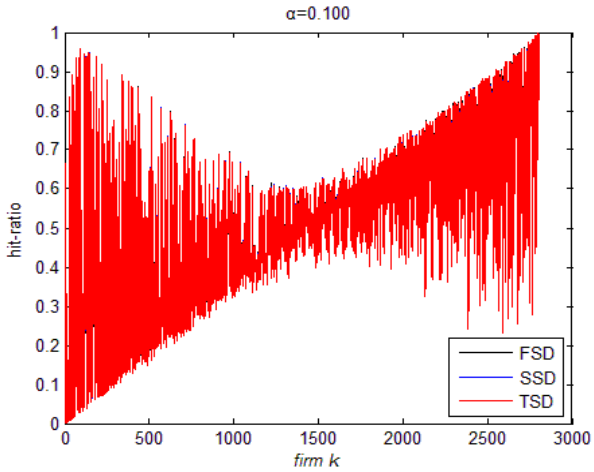

Figure 3. The three SD-trends of hit-ratios for all corporations. The shape of hit-ratio distribution in this figure will not change various α and please refer to the Appendix A.

The TSD line almost covers the other SD lines and they make us to believe that three SD have similar hit-ratios. The first inference in above figures makes this article become simpler and we just analyze FSD only. The second one, the corporation with a median of VaR seems no extreme hit-ratio and its value is about 0.5.

The hitting ratios resolve the problem of SD cannot rank. Another finding tells us the different critical value α will not change the shape of hit-ratio distributions. We show these results by figures in our appendix. The 'regular' distributions could make sure that SD-VaR criterion work well with various different α.

An interesting fact is that these corporations with smaller VaR seem to have higher 'average' hit-ratio than with higher VaR's. However, we find that the 'variances' of hit-ratio are increasing progressively when a corporation with lower or higher VaR. These variances are not welcome and we plan to decompose and retrieve them. They are the magical material for classification and so we include them into SD-VaR criterion.

3. Methods

A. Violation Analysis

In this section we try to check the capability to avoid the loss risk for a G-type corporation. This test will help us to confirm the important property of SD-VaR criterion that a G-type corporation has then lower risk on the investment. Therefore we refer to Berkowitz and O'brien (2002) and use their methods to show that G-type corporation has much lower risk than other types. According to the figures in Section 2, we know that the R-type corporations are not enough samples and not have a signification on Statistics to operate a violation test. Finally, we run SD-VaR criterion

and point out the G-type corporations on Dec/2011 and apply violation analysis on next month.

From the above tables we are easy to identify that whether α value is, the classify method for the combination of SD and VaR is effective significantly. The average values of VaR are smaller in G-type corporations than in T- or S-types. The violation refers to the excess of the VaR and a violation rate is the number of violation divides by the number of a special type corporation. Therefore, the mean violation is the average for violation. The violation rates and mean of G-type corporations are also lower than other types and these outcomes can show that G-type corporations have less risk of investments.

There are some decrement in violation mean, rates and their maximum various the critical values. Specially, they are significant on the G-type corporations and satisfy the feature of VaR. We refer to the proof on Section 2 and see the α condition require its value cannot too high. When the critical value is lower, the likelihood of SD-VaR criterion will higher. In this table, they show significantly lower risk of loss in G-type corporations.

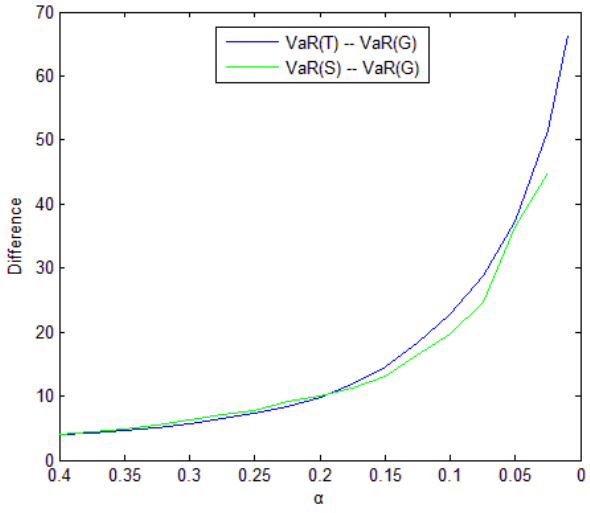

Figure 4. The differences between VaR(T), VaR(G) and VaR(S).

The differences in VaR between G-type and other types are strictly increasing various α. They express that the less risk of the G-type corporations are much superior to other types under a lower α than under a higher α. This fact responses the α requirements on the proof of ours study and a small α can confirm the probability of stochastic dominate a *i*-corporation for a G-type corporation is more than the T- or S-type corporation one.

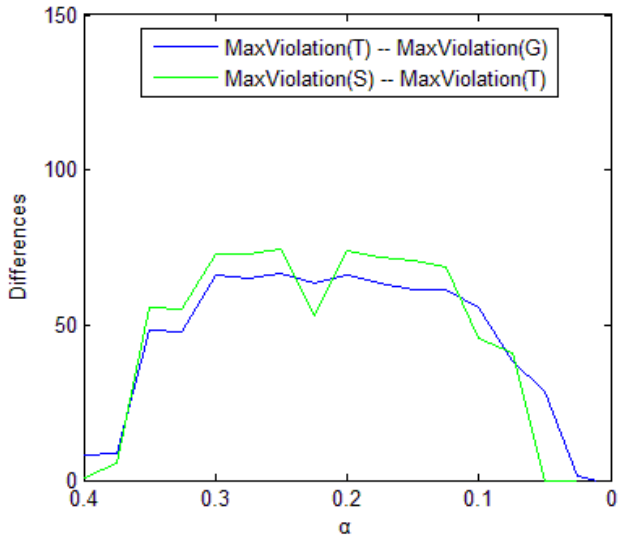

Figure 5. The two differences in the maximum violation. The blue line represents the difference in maximum violation between T-type and G-type. Another green line is expressed as the difference between S-type and T-type. It is very obvious that T- or S-type corporations are inferior to G-types as α between 0.1 and 0.3.

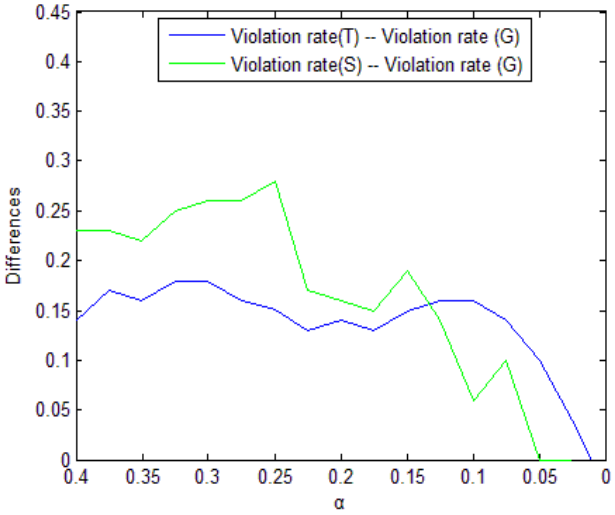

Figure 6. The violation-rate differences. The blue line reports the difference in violation rate between T-type and G-type. The green line describes the difference between S-type and G-type. After observing this figure, we find that the G-type corporation has the lowest violation rate than other two type's as $\alpha > 0.05$.

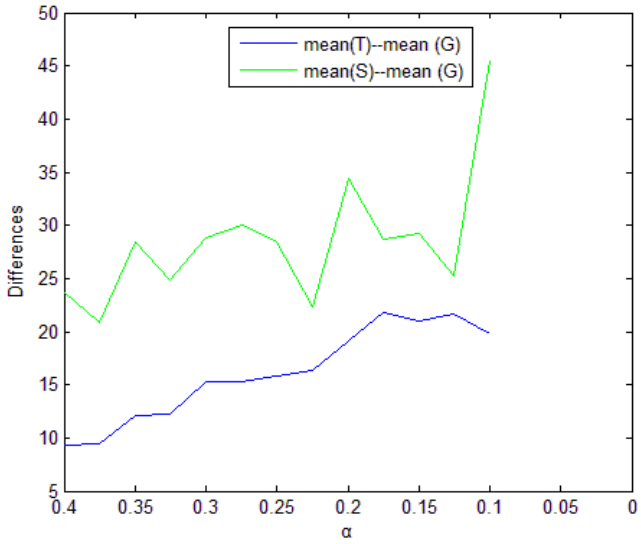

Figure 7. The two differences in violation mean between T-type and G-type, and S-type and G-type.

The number of violation for G-type corporations is only one violation as α less than 0.15 and his violation rates are lower than others. This table has sufficiently made

sure that G-type corporation is the lowest risk one.

B. Performance of the Profitability

On the Section 2, we adopt the *WER* to measure the chance to profit for 'individual' G-type and compare a G-type's *WER* with another individual *i*-corporation's.[5] But, we must show the profitability of 'a group of G-type corporations' on this section, therefore this article introduces the average of excess returns for G-type group to capture their chance to profit.

Table VIII

The profitability of G-type corporations

a. AER is the average of excess returns and its base is the market returns. **b**. Non-good corporation's AER is negative value of G-type corporation's AER

Selected corporations on Aug./2011	G-type corporation	Market avg. return
AER on Sep./2011	7.3661	1.9924
AER on Oct./2011	7.3158	2.0696
AER on Nov./2011	8.5250	2.0277
AER on Dec./2011	8.2742	1.9921

We follow Brown and Warner's (1980) market adjusted return model to examine the excess return of G-type corporation. They assume the expected return equals to market return and the excess return exactly is abnormal return in an equilibrium market. The table shows that the all returns of G-type corporations have obvious difference form market return. The G-type corporations have positive AER and outstanding profitability. The differences between AER for G-type corporation and

[5] Jensen index or Treynor index is also a good measure of profitability.

market return are more than 5% and easy (rough) to identify the existence of excess returns. The SD-VaR criterion point out the good corporations that have stable AER not only on next month, but also on future two, three and four month. The G-type corporation's capability of profit has a continuous property and help us to strengthen the prediction ability of SD-VaR criterion. The G-type is about five times average return of market and these outcomes satisfy the previous argument that our criterion can select the corporation with more profitability.[6]

Table IX

The proportions of corporations whose returns more than some special level

a. Return is the TRFM, **b**. α =0.1, **c.** the market return is about 2.03% on Dec/2011.

Selected corporations on Nov./2011	Ratio for G-type corporation	Ratio for Non-good corporation	Ratio of all corporations on Nov./2011
Returns >= 3% on Dec./2011	0.6429 (54/84)	0.0901 (462/5130)	0.0990 (516/5214)
Returns >= 4% on Dec./2011	0.5476 (46/84)	0.0604 (310/5130)	0.0683 (356/5214)
Returns >= 5% on Dec./2011	0.5000 (42/84)	0.0464 (238/5130)	0.0537 (280/5214)
Returns >= 6% on Dec./2011	0.4762 (40/84)	0.0351 (180/5130)	0.0422 (220/5214)

The table shows the 84 G-type corporation classified **on Nov/2011** has the higher probabilities of returns more than 3%, 4%, 5% and 6% respectively.[7] There is only 5.37% proportion of all corporations whose returns are more than 5%, but SD-VaR criterion's proportion is about 50%. When the market return is just about 2.03% and the G-type corporation has more possibility to possess the excess return after reading the proportions. These data supports and suggests that G-types have a larger

[6] Brown and Warner (1980, 1985): test statistics under the null hypothesis, event study.
[7] There are 87 G-type corporations on Dec/2011.

probability of its returns over 3% and more chance to profit on next month than other type. It is very easy to select out the 84 corporations have less risk of loss and more profitability from 5214 corporation using the SD-VaR criterion.

4. Expected performance

There are a lot of great assessing tools and their contributions are obvious on the literatures. We extract the complemented effects from two success ones and include them to create a fundamental idea. The SD-VaR criterion can point out a group of reliable corporations and we random sample from this group to determine a portfolio with an excess returns. The statuses of these good corporations we selected also have a persistency and predictable feature. These properties exactly satisfy the sufficiency conditions for a health corporation.

The dual assessing is necessary for selecting a portfolio. They like to have double filters and avoid including the error choices. This article adopts the most of corporation in COMPUSTAT database, there are more than five thousands corporation involving our analysis. Therefore, our results are comprehensive and the ideas of this criterion are so simplify and immediately.

We can adopt other criterion in place of VaR or SD filter, for example: CVaR, Jensen index, Treynor index, Sharpe ratio and so on. Even three filters are designed a new principle to select portfolio and these are the future issues. This article brings the dual assessing rule to balance risk and it just a fundamental conception. But, its outcomes exactly are the results we want.

REFERENCES

Altman, E. I., 1968, Financial ratios, discriminant analysis and the prediction of corporate bankruptcy, *Journal of Finance* 23, 589-609.

Basak, S. and A. Shapiro, 2001, Value-at-risk-based risk management: optimal policies and asset prices, *Review of Financial Studies* 14, 371-405.

Bawa, V. S., 1975, Optimal rules for ordering uncertain prospects, *Journal of Financial Economics* 2, 95-121.

Bawa, V. S., 1978, Safety-first, stochastic dominance, and optimal portfolio choice, *Journal of Financial Quantitative Analysis* 33, 255–271.

Berkowitz, J. and J. Obrien, 2002, How Accurate are value-at-risk models at commercial banks?, *Journal of Finance* 57, 1093-1111.

Brown, S. J. and J. B. Warner, 1980, Measuring security price performance, *Journal of Financial Economics* 8, 205-258.

Brown, S. J. and J. B. Warner, 1985, Using daily stock returns: the case of event studies, *Journal of Financial Economics* 14, 3-31.

Campbell, R., R. Huisman and K. Koedijk, 2001, Optimal portfolio selection in a value-at-risk framework, *Journal of Banking and Finance* 25, 1789-1804.

Davidsond, R. and J. Y. Duclos, 2000, Statistical inference for stochastic dominance and for the measurement of poverty and inequality, *Econometrica* 68, 1435-1464.

Dentcheva, D. and A. Ruszczynski, 2006, Portfolio optimization with stochastic dominance constraints, *Journal of Banking and Finance* 30, 433–451.

Duffie, D. and J. Pan, 1997, An overview of value at risk, *Journal of Derivatives* 4, 7-49.

Frydman, H., E. I. Altman and D. L. Kao, 1985, Introducing recursive partitioning for financial classification: the case of financial distress, *Journal of Finance* 40,

269-291.

Gupta, A. and B. Liang, 2005, Do hedge funds have enough capital? a value-at-risk approach, *Journal of Financial Economics* 77, 219–253.

Hadar, J. and W. R. Russell, 1969, Rules for ordering uncertain prospects, *American Economic Review* 59, 25-34.

Kopa, M. and T. Post, 2009, A portfolio optimality test based on the first-order stochastic dominance criterion, *Journal of Financial Quantitative Analysis* 44, 1103–1124.

Ma, C. H. and W. K. Wong, 2010, Stochastic dominance and risk measure: a decision-theoretic foundation for VaR and C-VaR, *European Journal of Operational Research* 207, 927-935.

Mas-Colell, A., M. D. Whinstion and J. R. Green, 1995, *Microeconomic Theory*, Oxford University Press, New York.

Post, T., 2003, Empirical tests for stochastic dominance efficiency, *Journal of Finance* 58, 1905–1932.

Rockafellar R. T. and S. Uryasev, 2002, Conditional value-at-risk for general loss distributions, *Journal of Banking and Finance* 26, 1443–1471.

Srinivasan, V. and Y. H. Kim, 1987, Credit granting: a comparative analysis of classification procedures, *Journal of Finance* 42, 665-681.

Whitmore, G. A., 1970, Third degree stochastic dominance, *American Economic Review* 60, 457–459.