Measuring Berry Phase of Entangled Photons Related to Orbital Angular Momentum

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Abstract: We demonstrate experimentally and theoretically that two-photon coincidence "ghost" imaging detection can reveal a generalized version of Berry phase related to orbital angular momentum. The round featureless signal light beam is generated by the spontaneous parametric down conversion and entangled to its counterpart while the Berry phase is added to the signal beam by varying its propagation vector to form a closed circuit.

OCIS codes: 270.4180, 190.4410

References and links

1. D. V. Strekalov, A. V. Sergienko, D. N. Klyshko, and Y. H. Shih, "Observation of 2-photon ghost interference and diffraction," Phys. Rev. Lett. **74**, 3600-3603 (1995).
2. M. D'Angelo, Y.-H. Kim, S. P. Kulik, and Y. Shih, "Identifying entanglement using quantum ghost interference and imaging," Phys. Rev. Lett. **92**, 233601 (2004).
3. A. F. Abouraddy, B. E. A. Saleh, A. V. Sergienko, and M. C. Teich, "Role of entanglement in two-photon imaging," Phys. Rev. Lett. **87**, 123602 (2001).
4. R. S. Bennink, S. J. Bentley, R. W. Boyd, and J. C. Howell, "Quantum and classical coincidence imaging," Phys. Rev. Lett. **92**, 033601 (2004).
5. D. N. Klyshko, "A simple method of preparing pure states of the optical-field, a realization of the Einstein, Podolsky, Rosen experiment and a demonstration of the complementarity principle," Usp. Fiz. Nauk **154**, 133-152 (1988).
6. V. Giovannetti, S. Lloyd, and L. Maccone, "Positioning and clock synchronization through entanglement," Phys. Rev. A **65**, 022309 (2002).
7. A. N. Boto, P. Kok, D. S. Abrams, S. L. Braunstein, C. P. Williams, and J. P. Dowling, "Quantum interferometric optical lithography: exploiting entanglement to beat the diffraction limit," Phys. Rev. Lett. **85**, 2733-2736 (2000).
8. M. D'Angelo, M. V. Chekhova, and Y. Shih, "Two-photon diffraction and quantum lithography," Phys. Rev. Lett. **87**, 013602 (2001).
9. M. V. Berry, "Quantal phase-factors accompanying adiabatic changes," Proc. R. Soc. Lond. A **392**, 45-57 (1984).
10. Y. Aharonov, and D. Bohm, "Significance of electromagnetic potentials in the quantum theory," Phys. Rev. **115**, 485-491 (1959).
11. R. Y. Chiao, and Y.-S. Wu, "Manifestations of Berry's topological phase for the photon," Phys. Rev. Lett. **57**, 933-936 (1986).
12. M. Segev, R. Solomon, and A. Yariv, "Manifestation of Berry's phase in image-bearing optical beams," Phys. Rev. Lett. **69**, 590-592 (1992).
13. E. J. Galvez, and C. D. Holmes, "Geometric phase of optical rotators," J. Opt. Soc. Am. A **16**, 1981-1985 (1999).
14. J. P. Torres, A. Alexandrescu, and L. Torner, "Quantum spiral bandwidth of entangled two-photon states," Phys. Rev. A **68**, 050301 (2003).
15. B. E. A. Saleh, A. F. Abouraddy, A. V. Sergienko, and M. C. Teich, "Duality between partial coherence and partial entanglement," Phys. Rev. A **62**, 043816-043815 (2000).
16. L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman, "Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes," Phys. Rev. A **45**, 8185-8189 (1992).
17. A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, "Entanglement of the orbital angular momentum states of photons," Nature **412**, 313-316 (2001).
18. E. J. Galvez, P. R. Crawford, H. I. Sztul, M. J. Pysher, P. J. Haglin, and R. E. Williams, "Geometric phase associated with mode transformations of optical beams bearing orbital angular momentum," Phys. Rev. Lett. **90**, 203901-203904 (2003).

19. A. T. O'Neil, and J. Courtial, "Mode transformations in terms of the constituent Hermite-Gaussian or Laguerre-Gaussian modes and the variable-phase mode converter," Opt. Commun. **181**, 35-45 (2000).
20. T. B. Pittman, Y. H. Shih, D. V. Strekalov, and A. V. Sergienko, "Optical imaging by means of two-photon quantum entanglement," Phys. Rev. A **52**, R3429-R3432 (1995).
21. G. Scarcelli, V. Berardi, and Y. Shih, "Can two-photon correlation of chaotic light be considered as correlation of intensity fluctuations?," Phys. Rev. Lett. **96**, 063602 (2006).
22. R. S. Bennink, S. J. Bentley, and R. W. Boyd, ""Two-photon" coincidence imaging with a classical source," Phys. Rev. Lett. **89**, 113601 (2002).
23. G. Scarcelli, A. Valencia, and Y. Shih, "Experimental study of the momentum correlation of a pseudothermal field in the photon-counting regime," Phys. Rev. A **70**, 051802 (2004).
24. T. Rom, T. Best, D. van Oosten, U. Schneider, S. Folling, B. Paredes, and I. Bloch, "Free fermion antibunching in a degenerate atomic Fermi gas released from an optical lattice," Nature **444**, 733-736 (2006).
25. S. Folling, F. Gerbier, A. Widera, O. Mandel, T. Gericke, and I. Bloch, "Spatial quantum noise interferometry in expanding ultracold atom clouds," Nature **434**, 481-484 (2005).

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1. Introduction

Entanglement is at the heart of quantum mechanics since its early development. Recently, the entanglement of photons generated by the spontaneous parametric down conversion(SPDC)[1-5] has been applied to quantum coincidence "ghost" imaging and suggested to many applications[6-8]. On the other hand, Berry phase[9], the geometric phase acquired by a system after following a closed circuit in Hamiltonian parameter space, plays a major role in many experiments related to quantum interference, one of the most known as the Aharonov-Bohm effect[10] of the electrons. First pointed out by Berry, this geometric phase can also be acquired by other particles. Later this idea was extended by Chiao[11] to include photons, whose spin state can be +1 or -1 with the Hamiltonian parameter space being replaced by the propagation vector $\vec{k}$ space. Following this track, Segev generalized Berry's geometric phase as the rotation of an image bearing light beam[12], whose angle of rotation about its propagation axis depends on the geometric phase determined by the solid angle subtended by the propagation vector $\vec{k}$ of the beam to go along a closed circuit[13].

Photons can carry not only spins but also orbital angular momentums(OAM) of multiple $\hbar$. Mostly, the OAM is represented by the mode number ℓ in the Laguerre-Gaussian (LG) mode, manifested in the helicity of the wave function of the light. If the total phase to go around a light beam about its center is $2\ell\pi$, the photon is carrying OAM of $\ell\hbar$. According to Berry's principle extended by Chiao, if the change ($\Delta\vec{k}$) of the propagation vector $\vec{k}$ of the light beam on its propagation path forms a closed circuit and encircles certain amount solid angle Ω, the photon will obtain a geometric phase of $-\ell\Omega$. Because the total phase going around the singular center of an ℓ mode is $2\ell\pi$, obtaining a phase $-\ell\Omega$ for such mode is equivalent to a rotation of angle Ω about its singular center. Therefore, the light beam composed of many different ℓ modes, though different ℓ modes obtain different amount of Berry phase, the entire beam will rotate about its axis by Ω. However, if the light beam is round, featureless, and unpolarized, a direct measurement cannot distinguish if there is a rotation of the light beam. In this Letter, we demonstrate experimentally and theoretically that the two-photon coincident detection can reveal the rotation related to the generalized Berry phase of such light beam.

2. Experiment

In the setup shown in Fig. 1, the argon ion laser beam, removed unwanted fluorescence by a prism, is pumped into a beta barium borate (BBO) crystal cut for type-II collinear degenerate spontaneous parametric down conversion (SPDC). The orthogonally polarized, entangled, signal and idler photons, at central wavelength $\lambda_s = \lambda_i = 2\lambda_p = 702.2$ nm, are separated from the pump by a dispersion prism. A polarization beam splitter (PBS) reflects the idler while transmits signal photons. The signal then passes through a double slit (a = 0.16 mm and d = 0.5 mm). The signal and idler are to be detected by the single photon detector modules D_1 and

D_2 , respectively. Each module contains a small square aperture (D_1=0.16 mm^2, D_2=0.09 mm^2), a filter centered at 700.6 nm with a 10 nm FWHM and a collecting lens. The distances from BBO to the double slit and from the double slit to D_1 are 24.5 cm and 80 cm, respectively. The idler propagates freely a distance of 103 cm from PBS to D_2. We perform the coincident detection by fixing D_1 in position and scanning D_2 for the two transverse dimensions (X_2 and Y_2) in an area of 7.5 mm by 7.5 mm with a scanning step of 0.3 mm. The outputs of D_1 and D_2 are connected to a coincidence counter with a time window of 5 nsec. Figure 2. (B) shows the coincident intensity. The period of the observed coincident interference pattern is consistent with the theoretical value 2.0 mm.

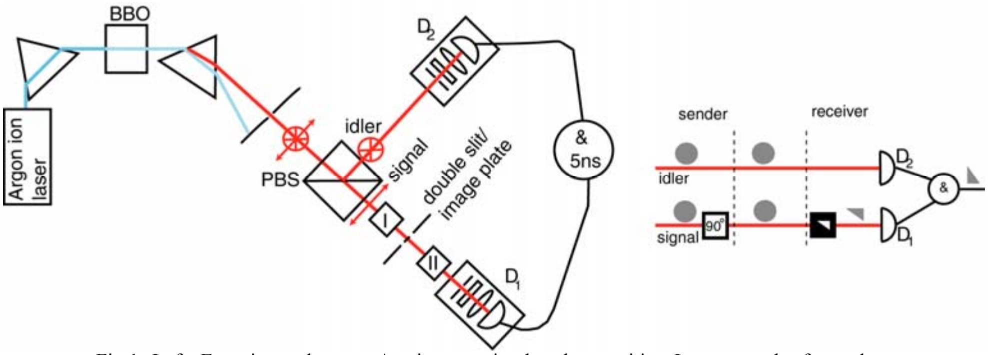

Fig.1. Left: Experimental setup. A mirror set is placed at position I to rotate the featureless signal beam, or at position II to rotate the pattern-bearing beam. For direct signal measurement, D_1 is scanned. For coincident measurement, D_1 is fixed while D_2 is scanned. Right: Sending information by featureless signal and idler light beams. The signal is rotated by some angle about its propagation axis and sent with the idler to the receiver. On the receiver side, an aperture shapes the signal. The receiver can obtain the rotation angle set by the sender.

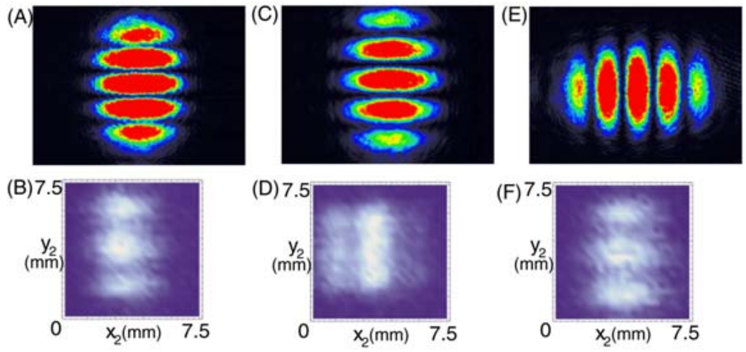

Fig. 2. (A), (C), and (E) show the interference patterns of the He-Ne laser beam, overlapped with the signal, to distinguish the orientation. (B), (D), and (F) show the coincident detection. Without the rotating mirrors, (A) and (B) have the same orientation. With the rotating mirrors at position I, the direct measurement (C) cannot reveal the rotation, while the coincident measurement (D) can. With the rotating mirrors at position II, the direct measurement (E) reveals such rotation, but the coincident measurement (F) cannot.

We then place a set of mirrors at position I in Fig. 1 that reflect the light a few times. The light that leaves the mirror set has the same propagation vector as it enters the mirror set. The change ($\Delta\vec{k}$) of the propagation vector $\vec{k}$ of the light beam on its propagation path therefore forms a closed circuit and encircles certain amount solid angle Ω. This is equivalent to adding Berry phase to the signal photons. Because of the large divergence of the round and featureless SPDC[1], we cannot observe the rotation caused by the added Berry phase directly. We therefore overlap a He-Ne laser beam with the signal beam and observe it by a CCD camera at the place of the detector module to discern the rotation. The interference pattern of the He-Ne laser beam is not rotated at all since it makes no difference to rotate a round featureless light beam [cf. Figs. 2(A) and (C)]. It means the added Berry phase cannot be revealed by the direct observation. We then conduct the coincident measurement for the SPDC light beams. The coincident interference pattern indeed rotate about $90°$ [cf. Figs. 2 (B) and (D)], i.e., the added Berry phase to the signal photons can be revealed by the "ghost" imaging.

On the other hand when the mirror set is at position II, the orientation of the interference pattern of the overlapping He-Ne laser beam [cf. Figs. 2(A) and (E)] is rotated about 90° as demonstrated by Segev[12]. Nevertheless, the coincident counting image reveals no rotation at all[cf. Figs. 2(B) and (F)].

2. Theory

The quantum state[14, 15] of the entangled photons is $\left|\Psi\right\rangle = \int d\bar{x}\Phi_p^+\left(\bar{x}\right)\hat{a}_s^+\left(\bar{x}\right)\hat{a}_i^+\left(\bar{x}\right)\left|0\right\rangle_s\left|0\right\rangle_i$. $\Phi_p^+\left(\bar{x}\right)$ is the profile of the pump, $\bar{x}$ the transverse coordinates, and $\hat{a}_{s(i)}^+$ the creation operator for the signal (idler). For a thorough analysis that can be applied to particles other than photons, in stead of a simpler geometric approach used to explain the original "ghost" imaging experiment [1], we detail the theory in the LG expansion [16]. In the complete LG basis, $\left|\Psi\right\rangle = \sum_{\ell_s,p_s}\sum_{\ell_i,p_i} C_{p_s,p_i}^{\ell_s,\ell_i}\left|\ell_s, p_s; \ell_i, p_i\right\rangle$, where $\left|\ell_{s(i)}, p_{s(i)}\right\rangle = \int d\bar{x} LG_{p_{s(i)}}^{\ell_{s(i)}}\left(\bar{x}\right)\hat{a}_{s(i)}^+(\bar{x})\left|0,0\right\rangle$ and $C_{p_s,p_i}^{\ell_s,\ell_i} = \int d\bar{x}\Phi_p^+\left(\bar{x}\right)\left[LG_{p_s}^{\ell_s}\left(\bar{x}\right)\right]^*\left[LG_{p_i}^{\ell_i}\left(\bar{x}\right)\right]^*$. Since the pump is the fundamental Gaussian mode carrying zero OAM[17], the state is reduced to $\left|\Psi\right\rangle = \sum_{\ell}\sum_{p_s,p_i} C_{p_s,p_i}^{\ell,-\ell}\left|\ell, p_s; -\ell, p_i\right\rangle$ and its density operator is $\hat{\rho} = \left|\Psi\right\rangle\left\langle\Psi\right| = \sum_{\ell,\ell'}\sum_{p_s,p_i}\sum_{p_s',p_i'} C_{p_s,p_i}^{\ell,-\ell}\left[C_{p_s',p_i'}^{\ell',-\ell'}\right]^*\left|\ell, p_s; -\ell, p_i\right\rangle\left\langle\ell', p_s'; -\ell', p_i'\right|$ due to the conservation of angular momentum.

For generality, we assume the signal and idler, respectively, are divided and transmitted through different linear systems with impulse response functions $h_s\left(\bar{x}_1,\bar{x}\right)$ and $h_i\left(\bar{x}_2,\bar{x}\right)$, where $\bar{x}_1$ and $\bar{x}_2$ are transverse coordinates on the output planes of the two systems, respectively. Berry's geometric phase, $-\ell\Omega$, obtained by each LG mode can be expressed [18, 19] as $\hat{T}\left|\ell, p\right\rangle = \exp\left(-i\ell\Omega\right)\left|\ell, p\right\rangle$, where Ω is the solid angle subtended by the propagation vector $\vec{k}$ of the beam to go along a closed circuit. With $\hat{T}$ at position II, the operators[15] at the detection planes of D_1 and D_2, respectively, are

$$\begin{cases} \hat{E}_1^+\left(\bar{x}_1\right) = \hat{T}\int d\bar{x} h_s\left(\bar{x}_1,\bar{x}\right)\hat{a}_s\left(\bar{x}\right) = \sum_{\ell,p}\sum_{\ell',p'}\exp\left[-i\ell\theta\right]\int d\bar{x} LG_p^{\ell}\left(\bar{x}_1\right)\left[LG_{p'}^{\ell'}\left(\bar{x}\right)\right]^*\gamma_{s(p,p')}^{(\ell,\ell')}\times\hat{a}_s\left(\bar{x}\right) \\ \hat{E}_2^+\left(\bar{x}_2\right) = \int d\bar{x} h_i\left(\bar{x}_2,\bar{x}\right)\hat{a}_i\left(\bar{x}\right) = \sum_{\ell,p}\sum_{\ell',p'}\int d\bar{x} LG_p^{\ell}\left(\bar{x}_2\right)\left[LG_{p'}^{\ell'}\left(\bar{x}\right)\right]^*\gamma_{i(p,p')}^{(\ell,\ell')}\times\hat{a}_i\left(\bar{x}\right) \end{cases}, \quad (1)$$

where we apply the completeness of $\mathrm{I} = \sum_{\ell,p}\left|\ell, p\right\rangle\left\langle\ell, p\right|$, and $\gamma_{s(i)(p,p')}^{(\ell,\ell')} = \int d\bar{x}_{1(2)} d\bar{x} h_{s(i)}\left(\bar{x}_{1(2)},\bar{x}\right)\left[LG_p^{\ell}\left(\bar{x}_{1(2)}\right)\right]^* LG_{p'}^{\ell'}\left(\bar{x}\right)$, referred to the transformation from $\left|\ell', p'\right\rangle$ to $\left|\ell, p\right\rangle$ by the impulse response functions h_s (h_i). Then the probability of coincidence of photons at the position $\vec{x}_1$ and $\vec{x}_2$ is

$$
\begin{aligned}
G(\vec{x}_1,\vec{x}_2) &= \langle 0,0|\hat{E}_1^{+}(\vec{x}_1)\hat{E}_2^{+}(\vec{x}_2)\hat{\rho}\hat{E}_2^{-}(\vec{x}_2)\hat{E}_1^{-}(\vec{x}_1)|0,0\rangle \\
&\propto \sum_{\ell,\ell'}\sum_{p_s,p_i}\sum_{p'_s,p'_i}\sum_{\ell_1,p_1}\sum_{\ell_2,p_2}\sum_{\ell'_1,p'_1}\sum_{\ell'_2,p'_2} C_{p_s,p_i}^{\ell,-\ell}\left[C_{p'_s,p'_i}^{\ell',-\ell'}\right]^* \exp\left(-i(\ell_1-\ell'_1)\Omega\right) LG_{p_1}^{\ell_1}(\vec{x}_1) \\
&\quad \times\left[LG_{p'_1}^{\ell'_1}(\vec{x}_1)\right]^* LG_{p_2}^{\ell_2}(\vec{x}_2)\left[LG_{p'_2}^{\ell'_2}(\vec{x}_2)\right]^* \gamma_{s(p_1,p_s)}^{(\ell_1,\ell)}\left[\gamma_{s(p'_1,p'_s)}^{(\ell'_1,\ell')}\right]^* \gamma_{i(p_2,p_i)}^{(\ell_2,-\ell)}\left[\gamma_{i(p'_2,p'_i)}^{(\ell'_2,-\ell')}\right]^*
\end{aligned}
\quad .
$$

The coincidence rate, when we fix D_1 and scan D_2, is then

$$
\begin{aligned}
\bar{P}(\vec{x}_2) &= \int_{\mathrm{P}_1} d\vec{x}_1 \times G(\vec{x}_1,\vec{x}_2) = [\cdots\cdots \underbrace{\int_{\mathrm{P}_1} d\vec{x}_1 \times LG_{p_1}^{\ell_1}(\vec{x}_1)\times\left[LG_{p'_1}^{\ell'_1}(\vec{x}_1)\right]^*}_{=\mathrm{A}_{p_1,p'_1}\delta(\ell_1-\ell'_1)} \cdots\cdots] \\
&\propto \sum_{\ell,\ell'}\sum_{p_s,p_i}\sum_{p'_s,p'_i}\sum_{\ell_1,p_1}\sum_{p'_1} C_{p_s,p_i}^{\ell,-\ell}\left[C_{p'_s,p'_i}^{\ell',-\ell'}\right]^* LG_{p_i}^{-\ell}(\vec{x}_2)\left[LG_{p'_i}^{-\ell'}(\vec{x}_2)\right]^* \mathrm{A}_{p_1,p'_1}\gamma_{s(p_1,p_s)}^{(\ell_1,\ell)}\left[\gamma_{s(p'_1,p'_s)}^{(\ell_1,\ell')}\right]^*
\end{aligned}
\quad , \qquad (2)
$$

where P_1 is the area of the aperture in front of D_1, located at the center of the signal beam and $h_i(\vec{x}_2,\vec{x})=1$, referred to vacuum, meaning $\gamma_{i(p,p')}^{(\ell,\ell')} \to \delta_{p,p'}\delta_{\ell,\ell'}$ has been applied. When the rotation is added before h_s, or $\hat{T}$ is at position I, $\hat{E}_1^{+\prime}(\vec{x}_1) = \int d\vec{x} h_s(\vec{x}_1,\vec{x})\hat{a}_s(\vec{x})\times\hat{T} = \sum_{\ell,p}\sum_{\ell',p'}\int d\vec{x} LG_p^{\ell}(\vec{x}_1)\left[LG_{p'}^{\ell'}(\vec{x})\right]^* \gamma_{s(p,p')}^{(\ell,\ell')}\hat{a}_s(\vec{x})\times\hat{T}$. One can get the coincidence rate as

$$
\begin{aligned}
\bar{P}'(\vec{x}_2) \;&\propto \sum_{\ell,\ell'}\sum_{p_s,p_i}\sum_{p'_s,p'_i}\sum_{\ell_1,p_1}\sum_{p'_1} C_{p_s,p_i}^{\ell,-\ell}\left[C_{p'_s,p'_i}^{\ell',-\ell'}\right]^*\left[\exp(-i\ell\theta)LG_{p_i}^{-\ell}(\vec{x}_2)\right] \\
&\quad \times\left[\exp(-i\ell'\Omega)LG_{p'_i}^{-\ell'}(\vec{x}_2)\right]^* \mathrm{A}_{p_1,p'_1}\gamma_{s(p_1,p_s)}^{(\ell_1,\ell)}\left[\gamma_{s(p'_1,p'_s)}^{(\ell_1,\ell')}\right]^*
\end{aligned}
\quad . \qquad (3)
$$

As we compare the results of Eq. (2) and Eq. (3), the Berry phase term disappears when $\hat{T}$ is at position II and appears when $\hat{T}$ is at position I. This is indeed what we observe in the experiment for the coincident detection.

4. Ghost Image Carrying Berry Phase

We also conduct the two-photon imaging[20] experiment with Berry phase added to the signal. We change the scheme and insert one lens after the PBS and place a image plate containing a triangular transparent area closely before the detecting module D_1, which already has a collecting lens. We scan transversely D_1 and obtain the direct image of the triangle [Fig. 3(A)].

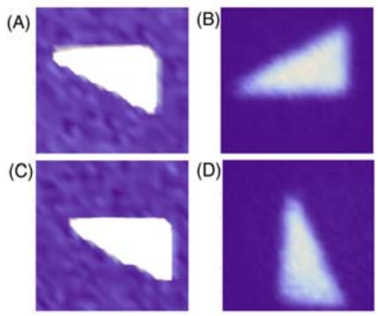

Fig. 3. Results of the imaging experiment: (A) and (C) show the images taken directly by scanning D_1. (B) and (D) show the coincident image. Without the rotating mirrors, (A) the direct and (B) the coincident measurements show an upside-down mirror inversion. With the rotating mirrors at position I, (C) the direct image shows no rotation, while (D) the coincident image shows a rotation.

We then scan D_2 and fix D_1, whose small aperture is removed to make it a bucket detector, and obtain the coincident image [Fig. 3(B)], of which the triangle is inverted upside down because the image mapped by one lens[20] is rotated by $180°$ and the reflection by the PBS for the idler makes a left-to-right mirror inversion. We then insert a four-mirror set[13], without introducing mirror inversion, to rotate the featureless signal light beam at position I. The direct measurement of the signal beams reveals no rotation [cf. Fig. 3 (A) and (C)]. However, the coincident image[cf. Figs. 3(B) and (D)] reveals a roughly $90°$ rotation. With this setup, by comparing the direct imaging and the coincident imaging, we can measure the rotation angle θ of the featureless signal beam and deduce that a signal photon of orbital angular momentum state ℓ has acquired a Berry phase $-\ell\theta$.

5. Conclusion and Discussion

One can transmit information as the rotation angle by round featureless signal and idler light beams as shown in the right part of Fig. 1. Or, one can remove the image mask and at the same time replace D_1 and D_2 by single photon detector arrays. When a coincident event is detected by the two arrays, one can retrieve the angle information transmitted by the sender by comparing the positions of the detectors on each array which register the incident of the photons. This way, with a single pair of entangled photons, one can transmit multi-bit information coded in the rotation angle set by the sender.

Recent researches demonstrated chaotic light[21-23] can be used as the light source for the two-photon coincident ghost imaging. With a careful theoretical examination, we find that similar results obtained here can be applied to the setup with chaotic light source because the degree of the mutual intensity coherence are both non-unity for the entangled photons and the chaotic light source. Therefore with similar setup, we can measure the rotation angle of a featureless chaotic light beam about its propagation axis and transmit information by the rotation angle of featureless chaotic light beams.

It is noticed that the spatial cross-correlation of cold fermionic atoms[24] and cold bosonic atoms[25] has already been measured by HBT type experiment. One might also extend the approach here to measure the Berry phase acquired by cold fermionic or bosonic atoms if a vector potential is present by realizing that the propagation vector and the magnetic field play the similar role in Berry's theory[9, 11].

This work was supported by National Science Council, Taiwan.